

Quadratic Functions: Understanding the Different Representations of Parabolas

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Introduction

Abstract: We explore the different ways of how quadratic functions are represented through equations, graphs, and tables. In order for mathematics students to get a deep understanding of how quadratic functions work, we must understand the advantages and disadvantages of each of the quadratic function representations, which are the standard form, vertex form, and factored form. Different methods of algebraic manipulation, such as factoring and completing the square, will be utilized. Furthermore, we explore how these functions can be represented on a graph, and then see how transformations affect the quadratic function. These various representations, methods, and topics are valuable for any mathematics student who is taking any form of an algebra course, as quadratic functions are used in future math courses, as well as in other types of school subjects such as Physics and Business Mathematics. By understanding the different representations of a quadratic function, as well as how it can be transformed, and then applied into different fields, will be very useful for education purposes.

Standard Form:
 $f(x) = ax^2 + bx + c$

Vertex Form:
 $f(x) = a(x - h)^2 + k$

Factored Form:
 $f(x) = a(x - r)(x - s)$

The Four Main Components of a Parabola

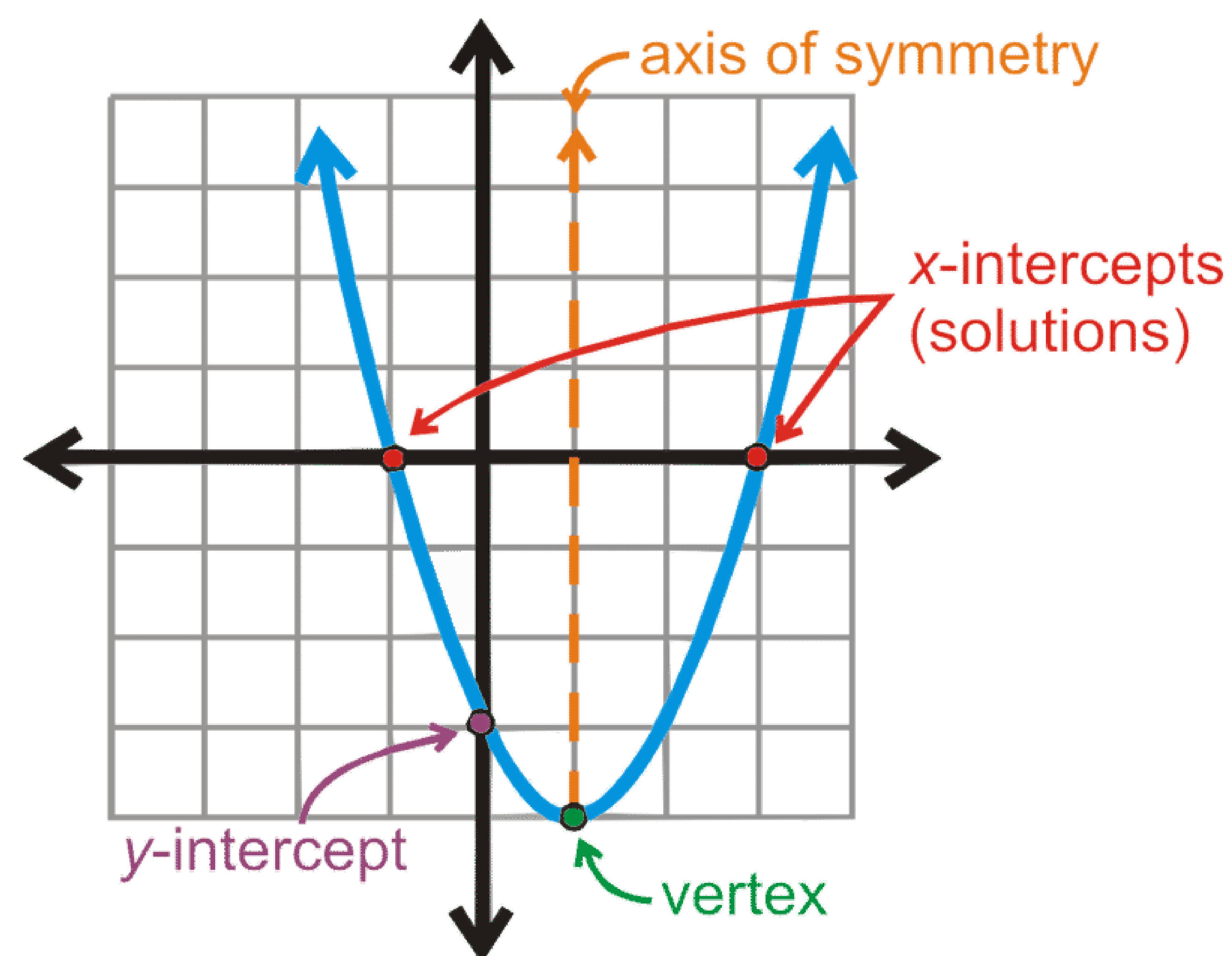
In order for students to have a good understanding of how quadratic functions work, they need to know all of the important components for graphing a quadratic function.

x-intercepts: These are where the parabola crosses the x-axis. These are also known as the *roots* or the *solutions* to the parabola.

y-intercept: This is where the parabola crosses the y-axis.

Vertex: This is the highest or lowest point of the parabola. It is also known as the turning point.

Axis of Symmetry: An imaginary line that divides the parabola into two identical parts.



Example

Consider the quadratic function $f(x)$ represented in the three forms below:

Standard Form:
 $f(x) = x^2 - 4x + 3$

Vertex Form:
 $f(x) = (x - 2)^2 + 1$

Factored Form:
 $f(x) = (x - 1)(x - 3)$

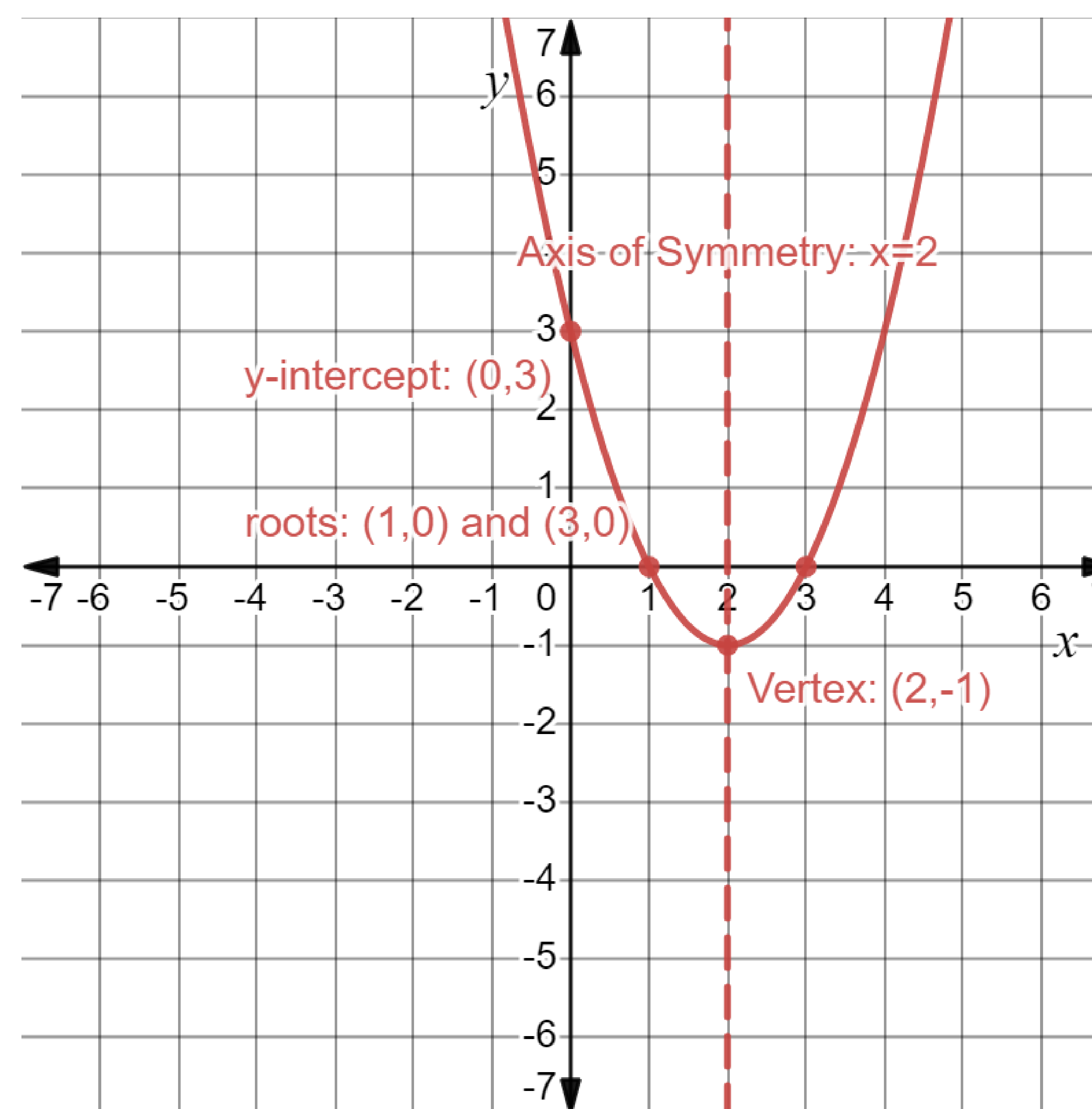
After getting familiar to the four components of a parabola, now students need to work on how to find them all.

Starting with the standard form, we can easily get the y-intercept, which is (0,3). We know this because by substituting $x = 0$ into the function, we are left with 3.

Moving on to the vertex form, we can then get the vertex and axis of symmetry. By analyzing the vertex form of the function, we can see that the vertex is (2,1). The axis of symmetry is therefore $x = 2$.

Finally, we have the factored form of the function. This is where we can find the x-intercepts of the parabola. By setting $f(x) = 0$, and solving for x by using the zero product property, we determine that the x-intercepts are $x = 1$ and $x = 3$.

With those four components, we now have our graph down below. In order to get the most accurate graph, we can substitute other x-values into one of the functions above to give us more (x, y) coordinates that we can plot on the graph.



Results

From this example, a great way to teach quadratic functions to students is to understand the advantages and the disadvantages of each form.

Standard Form

Advantages:

- You can determine the y-intercept, which is the 'c' value of the function.
- You can determine the vertex and axis of symmetry by using $(-\frac{b}{2a}, f(-\frac{b}{2a}))$ (although this is not as straight forward as looking at the vertex form.)

Disadvantages:

- In order to determine the x-intercepts, you will have to factor, and if the function is not factorable, you must use the quadratic formula.

Vertex Form

Advantages:

- You can determine the vertex and axis of symmetry easily.
- This is the best form for seeing how the parabola is being transformed (horizontal and vertical shifts, stretch/shrink, reflection across the x-axis, etc.)

Disadvantages:

- In order to determine the x-intercepts, you will have to solve for x (although this is not as difficult as determining the x-intercepts for the standard form.)

Factored Form

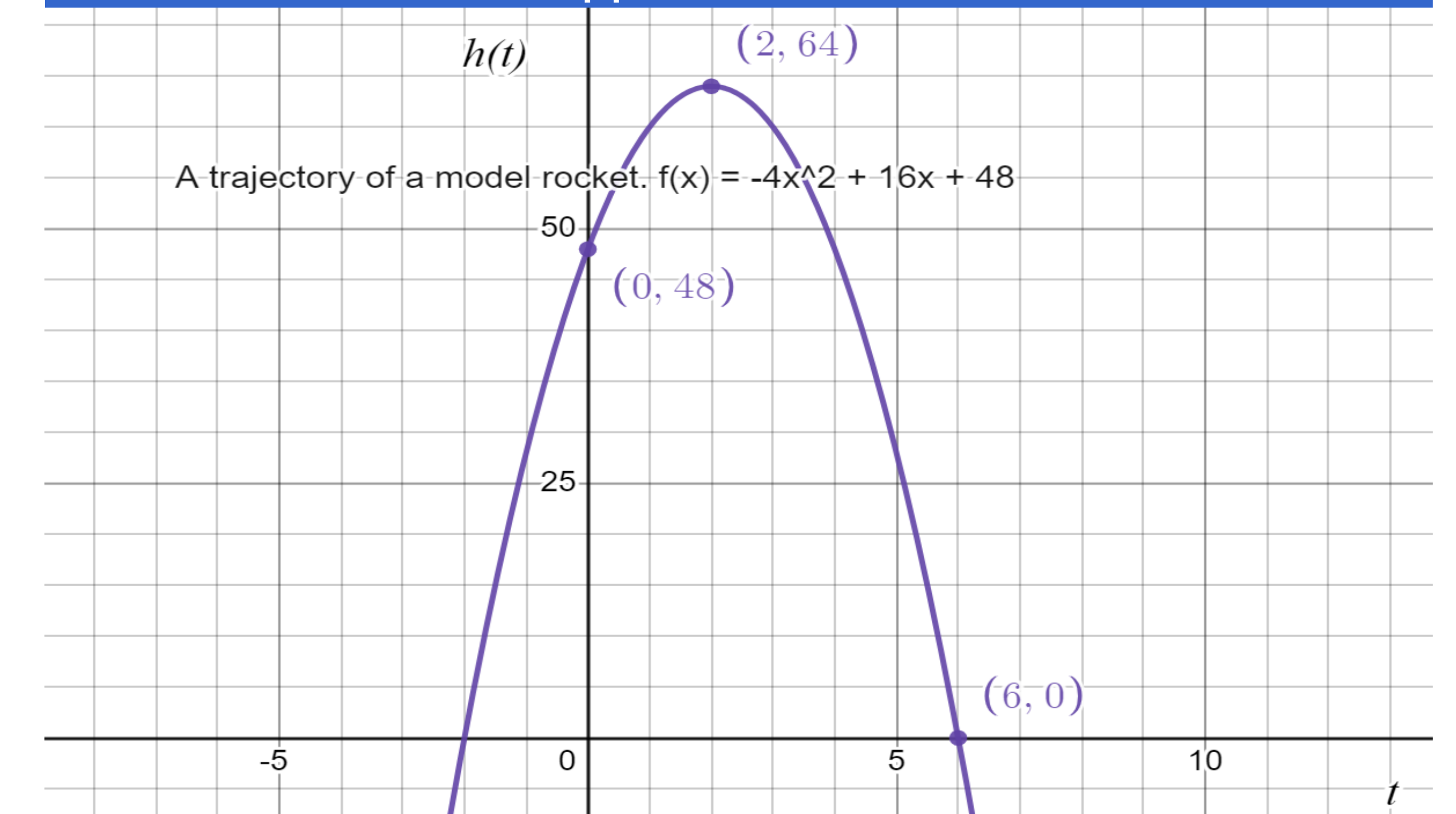
Advantages:

- You can determine the x-intercepts by using the zero product property.

Disadvantages:

- In order to determine the vertex and axis of symmetry, you must multiply the factors together to get the function in standard form, then use $(-\frac{b}{2a}, f(-\frac{b}{2a}))$.

Applied Problem



Bibliography

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